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NRL Memorandum Report 6160

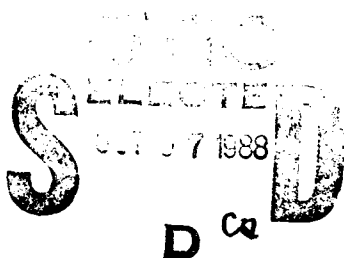
## Precise Interferometric Phase Determination

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August 30, 1988

AD-A200 186



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SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE				Form Approved GSA FPMR (41 CFR) 101-11.6	
1a REPORT SECURITY CLASSIFICATION UNCLASSIFIED			1b RESTRICTED MARKING		
2a SECURITY CLASSIFICATION AUTHORITY			3 DISTRIBUTION AVAILABILITY STATEMENT Approved for public release; distribution unlimited.		
2b DECLASSIFICATION/DOWNGRADING SCHEDULE					
4 PERFORMING ORGANIZATION REPORT NUMBER(S) NRL Memorandum Report 6160			5 MONITORING ORGANIZATION REPORT NUMBER		
6a NAME OF PERFORMING ORGANIZATION Naval Research Laboratory		6b OFFICE SYMBOL (If applicable)		7a NAME OF MONITORING ORGANIZATION	
6c ADDRESS (City, State, and ZIP Code) Washington, DC 20375-5000			7b ADDRESS (City, State, and ZIP Code)		
8a NAME OF FUNDING SPONSORING ORGANIZATION Defense Nuclear Agency		8b OFFICE SYMBOL (If applicable)		9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER	
8c ADDRESS (City, State, and ZIP Code)			10 SOURCE OF FUNDING NUMBERS		
			PROGRAM ELEMENT NO	PROJECT NO	TASK NO
			SUB-PROJECT ACCESSION NO		
11 TITLE (Include Security Classification) Precise Interferometric Phase Determination (UN)					
12 PERSONAL AUTHOR(S) Stamper, J.A.					
13a TYPE OF REPORT Interim		13b TIME COVERED FROM TO		14 DATE OF REPORT (Year, Month, Day) 1988 August 30	
15 PAGE COUNT 32					
16 SUPPLEMENTARY NOTES This work was supported by the Defense Nuclear Agency					
17 COSAT CODES			18 SUBJECT TERMS (Continue on reverse if necessary and identify by block number)		
FIELD	GROUP	SUB GROUP	Phase Interferometry		
			Fringes Interpolation		
			Non-uniform		
19 ABSTRACT (Continue on reverse if necessary and identify by block number)  A precise determination of phase shifts from interferometric fringes is necessary to accurately determine density structure. This is accomplished by a careful comparison of the pair of interferograms representing the initial and final phases. A step-by-step procedure is given for determining the interpolated phase shift from fringes crossing a given axial position of a medium with axial symmetry. Non-uniform final fringe spacing is assumed and accounted for in the interpolation. An example is included, as well as a listing of the computer program.					
20 DISTRIBUTION AVAILABILITY OF ABSTRACT <input checked="" type="checkbox"/> AVAILABLE UNLIMITED <input type="checkbox"/> AVAILABLE RESTRICTED <input type="checkbox"/> NOT AVAILABLE			21 ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED		
22a NAME OF RESPONSIBLE INDIVIDUAL John A. Stamper			22b TELEPHONE (Include Area Code) (202) 767-2683		22c OFFICE SYMBOL

DD Form 1473, JUN 86

Previous editions are obsolete  
S/N 0102-LF-014-6603

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## PRECISE INTERFEROMETRIC PHASE DETERMINATION

### I. INTRODUCTION

This report is primarily concerned with precise phase-shift determinations where the phase changes rapidly (steep-gradient region) or where the phase changes are small (outer, low-density region). The medium is assumed to have cylindrical symmetry. Two things are required: A precise knowledge of the initial phase (from a pre-shot interferogram) and a precise determination of the final phase by interpolating the shifted fringes of the data interferogram. Abel inversion of the phase-shift information may reveal important low-density structures which would be missed without using these techniques. One should note, in the Abel inversion procedure, that the density determination at a particular cylindrical radius does not depend on what happens at smaller radii. Non-uniform fringe spacing may be important in the steep-gradient regions and is included in the interpolation procedure.

The purpose of this report is to provide a general background and to set forth the method in a step-by-step procedure that can be put to practical use. The book-keeping details turned out to be rather involved and tricky. Thus, it seemed reasonable to include a detailed procedure for obtaining the necessary input from the pair of pre-shot and data interferograms as well as a listing of the computer program. The procedure is applied to a sample interferogram pair (Fig. (1)) obtained from a laser-produced plasma. The procedure includes preparing the interferogram pair for data extraction, tabulation of the data, and input to the computer program. The computed results (radius and phase shift) are given for the sample interferogram pair. I have attempted to make the program readable by the liberal use of comments to explain and de-limit.

The example illustrated in Fig. 1 is an experimentally-obtained interferogram pair from a plasma containing a laser-produced blast, wave ( $I \sim 9$  to 16). The method to be described provides useful phase-shift information for the region of the blast wave and the outer-most, low density region ( $I > 16$ ). However, the central region ( $I < 7$ ) shows such irregular phase variation that the method offers no advantage in that region.

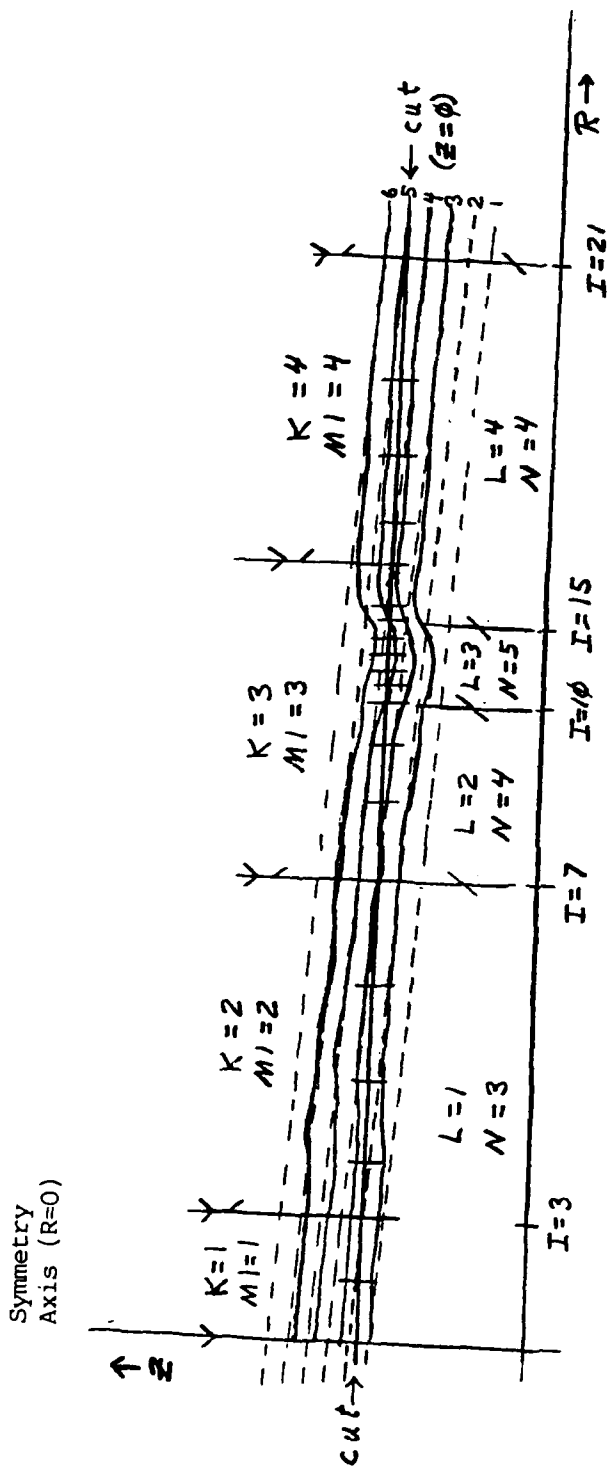


Fig. 1 Sample interferogram pair with 6 initial, or unshifted (dashed) and 4 shifted (continuous curves) fringes bounding the cut, showing intervals (I), spans (K), and crossing regions (L, between lower shifted fringe crossings). Also shown are the lower unshifted (M) and shifted (N) fringe numbers and the boundaries of intervals (—cut), spans (—), unshifted fringe crossings (—) and shifted fringe crossings (—).

## II. BASIC METHOD

We wish to determine the radial density profile at a given axial position  $Z$ , which is represented by the line  $Z = \text{constant}$  (called the "cut") in the  $R-Z$  plane of the interferogram pair. See Fig. 1. The final (shifted) fringes in the region of the cut are first divided into a sufficiently large number of radial "intervals" that the fringes are linear in each interval. Next, the region within the interval and between the two fringes bounding the cut is divided into 10 regions by introducing decimal "fringelets" which may be spaced non-uniformly to accommodate information in differing adjacent fringe spacings. The fringelets are illustrated in Fig. 2 with dashed lines but, in practice, are only treated numerically with the computer program. The non-uniform interpolation or fringelet spacing is described in the appendix. Each fringelet carries a known final phase. The intersections (radial position) of the fringelets with the cut, thus, have a known final phase. If no fringelet intersects the cut within the interval, the mid-radius of the interval is assigned the arithmetic average of the phases of the two fringelets which "straddle" the cut. Each intersection or straddle, which assigns a known final phase to a given radius within the interval, is called a "hit". The phase shift at a hit is obtained by subtracting the initial phase at the this radius.

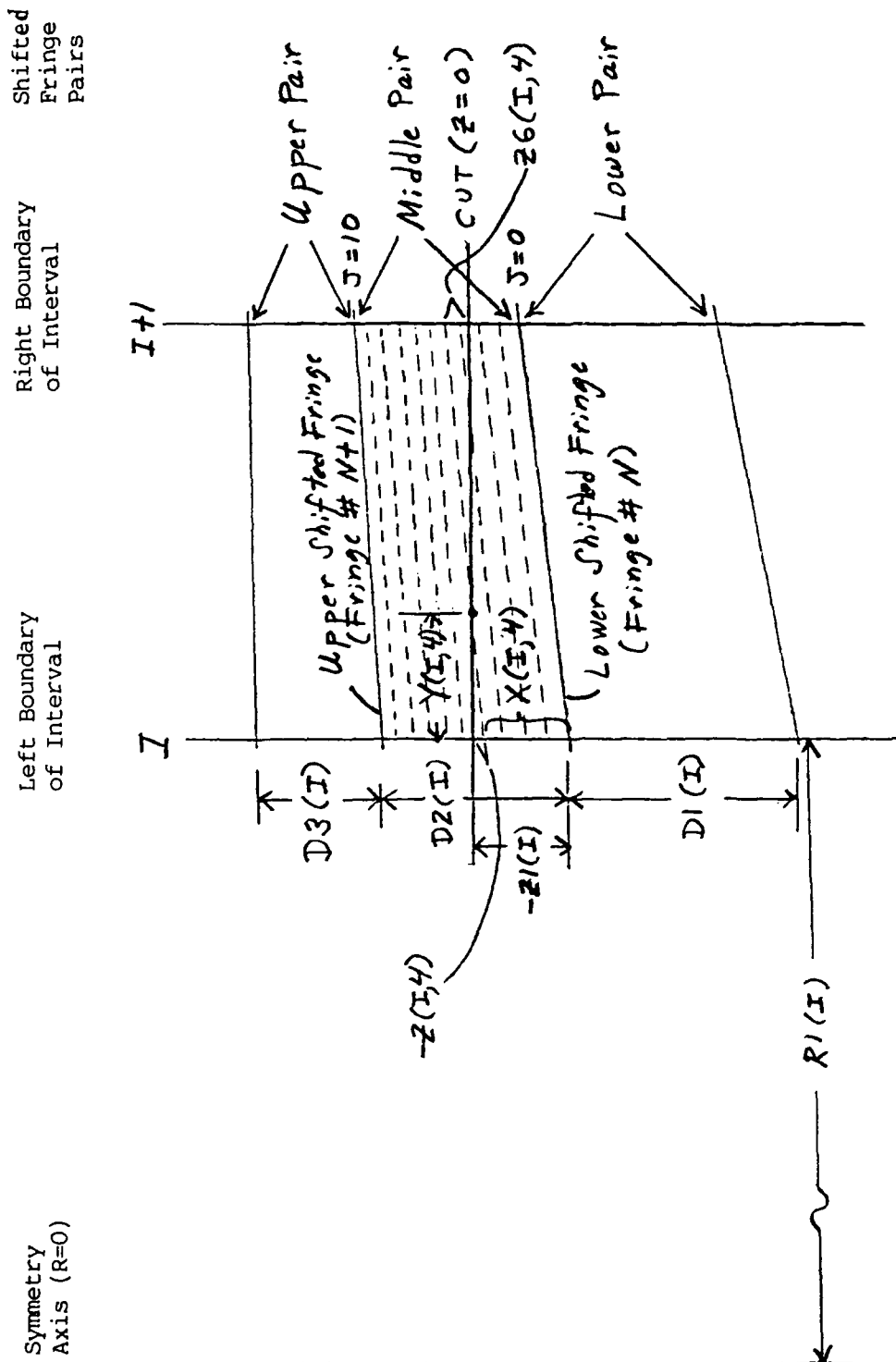


Fig. 2 Shifted (final) fringes within the  $I$ 'th interval, showing non-uniform decimal fringelets (dashed). Some  $J=4$  distances shown.

The initial phase is determined by the phases of the bounding initial fringes and by interpolation using the fractional distance (at the hit radius) of the cut from the lower fringe.

The implementation of the calculations involves a bit of bookkeeping and indexing, some of which is discussed here. Phase information is recorded by sequentially numbering the final and initial fringes. They must, of course, merge to the same values in the outer most, lowest density region of the medium. The initial fringes are usually close to straight lines but are divided in the region of the cut into a small number of radial "spans" in which the fringes are accurately linear. See Fig. 3. We are primarily interested in the pairs of initial and final fringes which bound the cut at a given radius. However, due to fringes crossing the cut, these pairs change with radius. Each interval  $I_1$  determines the span, with index  $K(I_1)$ , and the crossing region (for final fringes), with index  $L(I_1)$  of that interval. In turn,  $K(I_1)$  and  $L(I_1)$  determine, respectively, the bounding initial and final fringe pairs for that interval. The phase of the lower bounding fringe in an interval is determined by the fringe number:  $M_1(K(I_1))$  for initial fringes and  $N(L(I_1))$  for final fringes. Each hit (intersection or straddle of a shifted fringelet with the cut), with index  $I$ , is labeled with the interval number  $I_1(I)$  and fringelet number  $J_1(I)$ . See Fig. 2. A variable  $E(I)$  indicates whether a hit is an intersection

$(E(I)=0)$  or is a straddle ( $E(I)=.5$ ). The interpolated shifted fringe number at the hit is thus  $N(L(I1(I))) + (J1(I)-E(I))/10$ . The interpolated initial fringe number at the hit depends on the fraction  $F(I)$  into which the cut divides the fringe spacing at the hit radius. See Fig. 3. Here,  $F(I)=-W1(I)/(W2(I)-W1(I))$ . Thus, the interpolated initial fringe number is  $M1(K(I1(I)))+F(I)$ . The phase shift (in units of  $\pi$ ) is thus given by twice the difference in the shifted and initial fringe number at the hit.

$$P(I)=2*(N(L(I1(I)))+(J1(I)-E(I))/10-(M1(K(I1(I)))+F(I)))$$

The final result is the hit radius  $R(I)$  and phase shift  $P(I)$ , sorted for increasing radius.

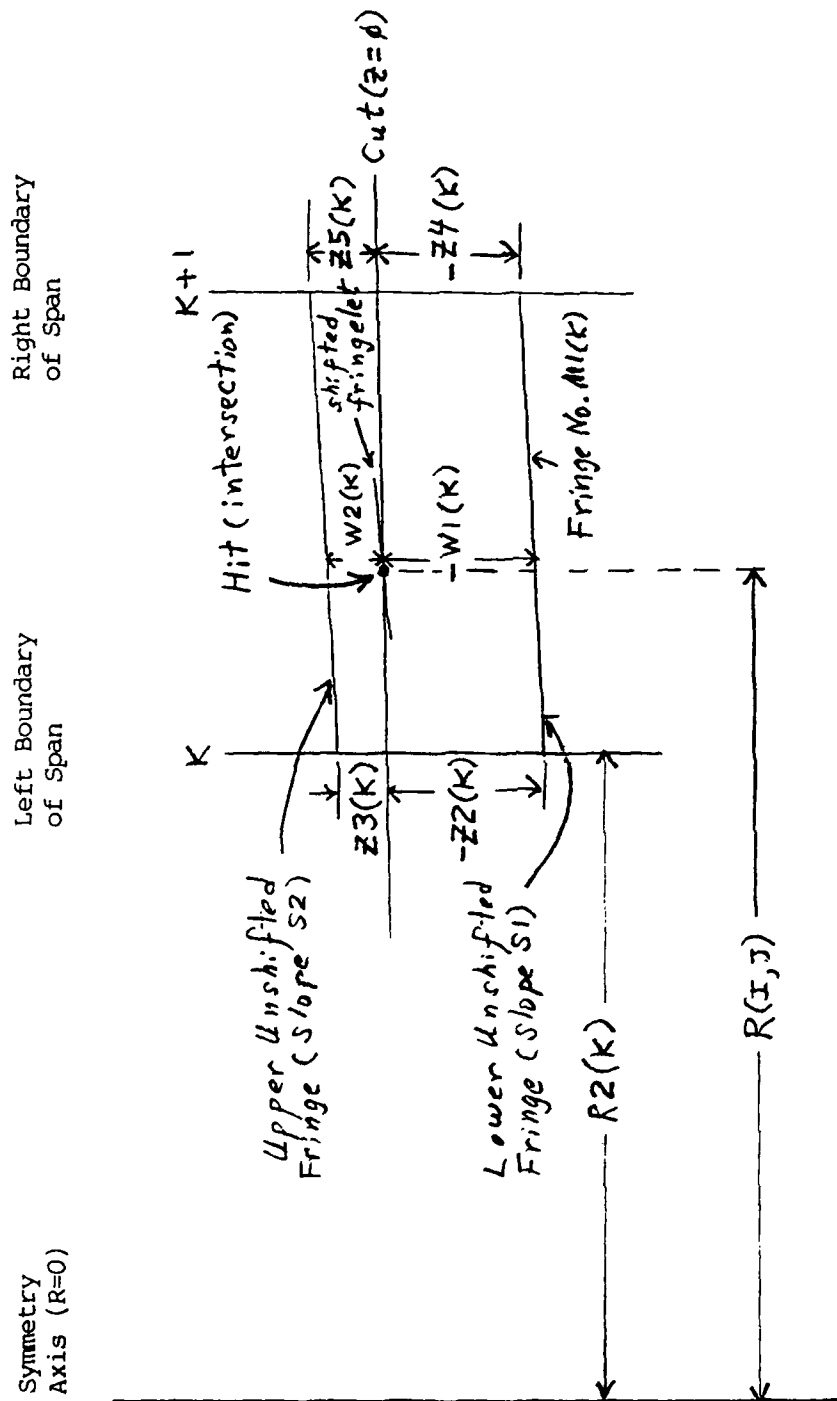


Fig. 3 Unshifted (initial) fringes within the  $K$ 'th span, showing one hit (intersection of shifted fringelet and cut).

### III. PROCEDURE AND EXAMPLE

A sample interferogram pair, prepared for extracting data, is shown in Fig.1. This is the superposition of two separate interferograms: The initial or pre-shot interferogram (shown dashed) and the final, shifted or data interferogram. The individual interferograms were projected and the fringes carefully aligned to each other for reference positions and merging to the same outer fringes. This happened to be the interferogram pair from which the procedure was developed and was not the optimum pair of interferograms for accurate application of the method. It may be helpful to refer to Figs. 2 and 3, showing separately information on the initial and final interferograms.

We first draw in the cut at the desired Z-position, taking into consideration the requirement of cylindrical symmetry. We next divide the initial fringes into a small number (4, in the example) of radial regions or spans such that: (1) Initial fringe crossings only occur at span boundaries and (2) the fringes are accurately linear within a span. The symmetry axis ( $R=0$ ) and outer radius (of fringe shifting) are taken as span boundaries. Spans are designated (K) by their left, or inner boundary. Span boundaries (vertical lines) are extended into the region above the cut and the following symbols used to aid recognition:  $\downarrow$  (for span boundary),  $\uparrow$  (for initial fringe crossing the cut).

Each span is labeled with the span index (K) and lower initial fringe number (M1) of fringes bounding the cut. It is also helpful to note the span boundary radii at each span boundary.

Information about the shifted fringes is indicated in the region well below the cut. First, locate all shifted fringe crossings, pass a vertical line through each crossing and indicate with a slanted dash ( $\diagdown$ ) that it is a crossing. Label each crossing region (below the cut) with the crossing index L and lower shifted fringe number N e.g., ( $\diagdown$   $\begin{smallmatrix} L=2 \\ N=4 \end{smallmatrix}$   $\diagdown$ ). Next, draw in (short vertical lines) interval boundaries such that: (1) all fringe crossings (initial or shifted) and span boundaries are interval boundaries, and (2) the final (shifted) fringes are linear in each interval. Intervals are designated (I) by the left or inner boundary. The symmetry axis ( $I=1$ , where  $R=0$ ) and outer radius ( $I=I_0+1$ , where  $I_0$  is the number of intervals) are also interval boundaries. The interval number of the shifted fringe crossings are indicated below the crossing (e.g.,  $I=7$ ).

We are now in a position to organize the information in tabular form, on data sheets, so that it can be input to the program. At the top of the first sheet, note the identifying (e.g., shot) number, the magnification (needed in Abel inversion) and the number of intervals ( $I_0$ ), spans ( $K_0$ ) and shifted fringe crossings ( $N_0$ ). Since lower fringe numbers (used in

calculating phase) depend directly on the span (initial) and the crossing region (shifted) while information is input for each interval, it is useful to relate each interval index to a span and crossing index. This cross-indexing is calculated by inputs from Table I. Next we record (in Table II) the lower fringe numbers.

The next step is to make detailed measurements of radial and axial distances and to organize this data into tabular form. In the shifted fringes, it is necessary to measure and record (in Tables III and IV), for each interval, the interval index  $I$ , the radial position  $R1(I)$  of the left boundary, the axial position (negative)  $Z1(I)$  of the lower fringe from the cut, the fringe slope  $S\emptyset(I)=(dZ/dR)/ABS(dZ/dR)$ , and the lower, central, and upper fringe-pair spacings  $D1(I)$ ,  $D2(I)$ ,  $D3(I)$ . The distances are all measured in centimeters. The fringe slope is  $\emptyset$  for the cut and +1 or -1 according as  $Z$  increases or decreases with  $R$  for a fringe. CAUTION: At a fringe crossing, we can have either a  $J=\emptyset$  intersection (where  $Z1(I)=\emptyset$ , since  $S\emptyset(I)=-1$ ) or a  $J=1\emptyset$  intersection (where  $Z1(I)=-D2(I)$ , since  $S\emptyset(I)=1$ ).

The shifted fringe input is first recorded, in Table III for the outermost radius (where  $I=I\emptyset+1$ ). The shifted fringe input at all left interval boundaries ( $I=1$  to  $I\emptyset$ ) is recorded in Table IV.

The input data for the initial fringes are recorded at each span. This includes the radius  $R2(K)$  of the (left-hand) span boundary, the axial positions (negative) of the left-side  $Z2(K)$  and right-side ( $Z4(K)$ ) of the lower fringes and the axial positions (positive) of the left-side  $Z3(K)$  and the right-side  $Z5(K)$  of the upper fringes. The values are recorded (Table V) for the left boundaries of the individual spans.

The detailed shifted-fringe information for each interval (Table IV) is entered, before the program is run, into DATA statements near the end of the program. The data for cross-indexing (Table I), lower fringe identification (Table II), outer radius information for shifted fringes (Table III), and the initial fringe information (Table V) are entered during the running of the program, as requested by input statements.

Interferometry Data Sheet - 1  
(Enter as requested by program, during run)

Shot No. 14760  
IØ=21

KØ=4

Mag. M=19  
NØ=4

Table I. Cross-Indexing: The intervals (left interval boundaries) I2 which occur at interior fringe crossings and the intervals (left boundaries) I3 which occur at interior span boundaries are:

Crossing Index L-1	Interval Index I=I2(L-1)	Span Index K-1	Interval Index I=I3(K-1)
1	7	1	3
2	10	2	7
3	15	3	18

Table II. The lower (shifted/initial) fringe numbers vs. (region/span) index (L/K).

Index L	Fringe # N(L)	Index K	Fringe # M1(K)
1	3	1	1
2	4	2	2
3	5	3	3
4	4	4	4

Table III. Shifted fringe input for the outermost radius (IØ+1).

R1=15.3  
D1= .30

Z1=0  
D2=.30

SØ=-1  
D3=.30

# Interferometry Data Sheet - 2

(Enter in Data Statements; 1 interval per statement)

Table IV. At the left (inner) boundary of each interval, the index  $I$ , radius  $R1(I)$ , axial position of lower fringe  $Z1(I)$ , fringe slope  $S0(I)$ , and lower, central and upper fringe-pair spacings  $D1(I)$ ,  $D2(I)$ ,  $D3(I)$  are:

$I$	$R1(I)$	$Z1(I)$	$S0(I)$	$D1(I)$	$D2(I)$	$D3(I)$
1	0	-.19	-1	.40	.40	.35
2	0.87	-.22	-1	.38	.38	.38
3	1.78	-.30	-1	.40	.40	.40
4	2.52	-.26	-1	.38	.38	.32
5	3.69	-.23	-1	.36	.38	.34
6	5.02	-.25	-1	.36	.35	.30
7	6.50	0	-1	.37	.32	.32
8	7.10	-.08	-1	.38	.30	.32
9	8.40	-.14	-1	.38	.30	.28
10	9.02	0	-1	.28	.30	.30
11	9.28	-.02	-1	.30	.32	.33
12	9.45	-.07	-1	.31	.33	.34
13	9.67	-.13	0	.27	.33	.34
14	9.90	-.12	1	.24	.30	.34
15	10.18	-.25	1	.20	.25	.25
16	10.37	-.14	1	.20	.24	.33
17	10.69	-.10	1	.29	.22	.32
18	10.92	-.10	0	.30	.22	.33
19	11.51	-.13	-1	.32	.27	.29
20	12.52	-.17	-1	.30	.28	.29
21	13.58	-.20	-1	.29	.29	.30

Interferometry Data Sheet - 3  
(Enter or requested by Program, during run)

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Table V. Initial Fringe Information. The Radii and Axial Position at the Span Boundaries are:

Span Index K	Radius R2(K)	Lower Fringe		Upper Fringe	
		Z2(K)	Z4(K)	Z3(K)	Z5(K)
1	0	-.18	-.30	.12	0
2	1.78	0	-.31	.31	0
3	6.50	0	-.30	.30	0
4	10.92	0	-.29	.29	0

#### IV. Computer program with data input for the interferogram pair shown in Fig. 1.

```

10 PRINT "NIFOI, A PROGRAM USING NON-UNIFORMLY-SPACED INTERFERENCE FRINGE DECIM-
MAL- INTERPOLATION. DECIMAL (INTERPOLATED) FRINGES ARE CALLED FRINGELETS."
20 PRINT "THE RADIAL VARIATION OF PHASE IS DETERMINED AT A FIXED AXIAL POSITION
OR LINE, CALLED THE CUT."
30 INPUT "SHOT OR IDENTIFYING NUMBER AND MAGNIFICATION";ID,MG
40 PRINT "THE SHIFTED FRINGES ARE MEASURED FOR MANY RADIAL INTERVALS WITH IN-
DEX I"
50 PRINT "THE INITIAL FRINGES ARE TAKEN AS LINEAR FOR A FEW RADIAL SPANS
WITH INDEX K(I)"
60 PRINT "ALL SHIFTED AND INITIAL FRINGE CROSSINGS ARE TAKEN AT INTERVAL BOUN-
DARIES. ALL UNSHIFTED FRINGE CROSSINGS ARE AT SPAN BOUNDARIES."
70 INPUT "THE NUMBER I0 OF RADIAL INTERVALS IS";I0
80 INPUT "THE NUMBER K0 OF RADIAL SPANS IS";K0
90 INPUT "THE NUMBER N0 OF SHIFTED FRINGE CROSSINGS IS";N0
100 DIM A(30),B(30),C(30),D(30),E(30),F(30),G(30),H(30),I(30),J(30),
K(30),L(30),M(30),N(30),O(30),P(30),Q(30),R(30),S(30),T(30),
U(30),V(30),W(30),X(30),Y(30),Z(30)
110 DIM Z(10),Z1(10),Z2(10),Z3(10),Z4(10),Z5(10)
120 DIM V(50,11),X(50,11),Y(50,11),Z(50,11),Z1(50,11),Z2(50,11),
Z3(50,11),Z4(50,11),Z5(50,11)
130 -----
140 -----SHIFTED FRINGE CALCULATIONS-----
150 -----
160 -----INDEXING TO INTERVAL AND NUMBERING-----
170 PRINT "INTERVALS ARE LABELED BY THEIR INNER OR LEFT BOUNDARY"
180 PRINT "THE SHIFTED FRINGE CROSSINGS ARE LOCATED AT INTERVAL BOUNDARIES I0+1
AND AT I="
190 -----I = CROSSING INDEX L-1 FOR FOLLOWING LOOP-----
200 FOR I=1 TO N0-1
210 INPUT "I0+L-1=";I0(I)
220 NEXT I
230 L=0
240 ----- I=INTERVAL INDEX FOR FOLLOWING LOOP-----
250 U1 =1
260 FOR I=1 TO I0(N0-1)
270 IF I(I0(U1)) THEN 290
280 U1=U1+1
290 L(I)=U1
300 NEXT I
310 FOR I=I0(N0-1) TO I0
320 L(I)=N0
330 NEXT I
340 PRINT "THE SHIFTED (LOWER) FRINGE NUMBERS ARE"
350 FOR L=1 TO N0
360 INPUT "N(L)=";N(L)
370 NEXT L
380 PRINT "THE (LOWER) FRINGE-SLOPE VARIABLE S0(I) IS (dZ/dR)/ABS(dZ/dR)"
390 -----
400 -----READ AND INPUT DATA AT EACH INTERVAL-----
410 PRINT "THE INTERVAL INDEX I, RADIAL (LEFT) BOUNDARY POSITION R(I), (NEG.)
AXIAL POSITION Z(I),FRINGE-SLOPE VARIABLE S0(I) AND THE LOWER, CENTRAL AND
UPPER FRINGE-PAIR SPACINGS D1(I),D2(I),D3(I) ARE READ IN FOR EACH DATA LINE."
420 "*****Z1(I) IS THE LEFT-SIDE AXIAL POSITION OF THE LOWER SHIFTED FRINGE.FOR
EACH J = 10 INTERSECTION (WITH S0(I)=1), Z1(I)=-D2(I), RATHER THAN 0.*****"
430 FOR I=1 TO I0
440 READ I,R(I),Z(I),S0(I),D1(I),D2(I),D3(I)
450 NEXT I
460 INPUT "AT THE OUTER RADIUS (I=I0+1) R1,Z1,S0,D1,D2,AND D3 ARE";R1(I0+1),
Z1(I0+1),S0(I0+1),D1(I0+1),D2(I0+1),D3(I0+1)
470 -----
480 -----FIND AXIAL FRINGELET POSITIONS AT INTERVAL BOUNDARIES-----
490 FOR I = 1 TO I0-1
500 G(I) = (D2(I) - D1(I))/180
510 H(I) = (D2(I) - D1(I))/180

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520 A(I) = (D1(I) + D2(I))/20
530 B(I) = (-19*A(I) + 9*H(I))/8
540 C(I) = (3*B(I) - H(I))/8
550 NEXT I
560 FOR I=1 TO 10+1
570 X(I,0)=0
580 Z(I,0)=Z1(I)
590 NEXT I
600 FOR I = 1 TO 10+1
610 FOR J = 1 TO 9
620 U(I,J) = B(I) + (2*J - 1)*C(I)/3
630 X(I,J) = J*(A(I) + (J-1)*U(I,J)/2)
640 Z(I,J) = Z1(I) + X(I,J)
650 NEXT J
660 Z(I,10) = Z1(I) + D2(I)
670 NEXT I
680 -----
690 -----RIGHT-HAND INTERVAL BOUNDARY VALUES-----
700 FOR I = 1 TO 10
710 FOR J = 1 TO 10
720 IF Z1(I+1) = 0 OR Z1(I+1) = -D2(I+1) THEN 750 ELSE 730
730 X(I+1,J) = X(I+1,J)
740 GOTO 790
750 IF Z1(I+1) = 0 THEN 760 ELSE 780
760 X(I+1,J) = (D1(I+1)/D2(I+1))*X(I,J)
770 GOTO 790
780 X(I+1,J) = (D2(I+1)/D2(I+1))*X(I,J)
790 IF Z1(I+1) = 0 THEN 800 ELSE 820
800 Z6(I,J) = X(I,J) - D1(I+1)
810 GOTO 850
820 IF Z1(I+1) = -D2(I+1) THEN 830 ELSE 850
830 Z6(I,J) = X(I,J)
840 GOTO 860
850 Z6(I,J) = Z1(I+1) + X(I,J)
860 NEXT J
870 NEXT I
880 -----
890 -----FIND HITS (FRINGELET INTERSECTIONS WITH OR STRADDLES OF CUT)-----
900 -----RADIAL POSITION IN INTERVAL IS-----
910 FOR I = 1 TO 10
920 FOR J= 1 TO 9
930 Y(I,J) = -Z(I,J)*(R1(I+1) - R1(I))/(Z6(I,J) - Z(I,J))
940 NEXT J
950 NEXT I
960 H = 1
970 FOR I = 1 TO 10
980 IF Z1(I) = 0 THEN 990 ELSE 1010
990 J = 0
1000 GOTO 1180
1010 IF Z1(I) = -D2(I) THEN 1020 ELSE 1040
1020 J = 10
1030 GOTO 1180
1040 J = 1
1050 IF ABS(Z6(I,1) - Z(I,1))<.001 THEN 1070 '--FRINGELET 1 PARALLEL TO CUT--
1060 IF Z6(I,1)>Z(I,1) AND Z6(I,1)>0 THEN 1180
1070 FOR J = 2 TO 9
1080 IF ABS(Z6(I,J) - Z(I,J))<.001 THEN 1110 '--FRINGELET J PARALLEL TO CUT--
1090 IF J=3 GOTO 1110
1100 IF Y(I,J)>0 AND Y(I,J)<R1(I+1)-R1(I) THEN 1130 '--INTERSECTION TEST--
1110 IF Z(I,J)/Z(I,J-1)<0 AND Z6(I,J)/Z6(I,J-1)<0 THEN 1260 '--STRADDLE TEST--
1120 NEXT J
1130 NEXT I
1140 GOTO 1220
1150 -----
1160 -----COUNT AND LABEL HITS-----

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1170 -----INTERSECTION COUNTER FOLLOWS-----
1180 H = H + 1
1190 E(H) = 0
1200 I(H) = I
1210 J(H) = J
1220 IF J=0 THEN 1040 ELSE 1230
1230 IF J = 10 THEN 1040 ELSE 1240
1240 IF J=1 THEN 1070 ELSE 1120
1250 -----STRADOLE COUNTER FOLLOWS-----
1260 H = H + 1
1270 E(H) = .5
1280 I(H) = I
1290 J(H) = J
1300 GOTO 1120
1310 -----OUTER INTERSECTION-----
1320 I = 10
1330 IF Z(I,10) < Z(I,10-1) THEN 1340 ELSE 1350
1340 J = 10
1350 GOTO 1370
1360 J = 0
1370 H = H + 1
1380 E(H) = 0
1390 I(H) = I
1400 J(H) = J
1410 PRINT "THE NUMBER H OF HITS IS",H
1420 DIM P(H),P(H),R(H),RR(H),B(H),S(H),SS(H),L(H),WZ(H)
1430 -----CENTER (R=0) HIT-----
1440 R(I) = 0
1450 I(I) = 1
1460 E(I) = 0
1470 IF Z(I,1) = 0 THEN 1480 ELSE 1500
1480 J(I) = 0
1490 GOTO 1640
1500 FOR J = 1 TO 9
1510 IF Z(I,J) = 0 THEN 1520 ELSE 1540
1520 J(I) = J
1530 GOTO 1640
1540 NEXT J
1550 E(I) = .5
1560 IF Z(I,1) > 0 THEN 1570 ELSE 1590
1570 J(I) = 1
1580 GOTO 1640
1590 FOR J = 2 TO 10
1600 IF Z(I,J)/Z(I,J-1) < 0 THEN 1610 ELSE 1630
1610 J(I) = J
1620 GOTO 1640
1630 NEXT J
1640 -----
1650 -----FIND RADII OF FRINGELET CROSSINGS-----
1660 FOR I = 2 TO H - 1
1670 IF Z(I(I),J(I)) <> 0 THEN 1680 ELSE 1730
1680 IF E(I) = 0 THEN 1690 ELSE 1710
1690 R(I) = R(I(I)) - Z(I(I),J(I))*(R(I(I)) + 1) - R(I(I))/(Z6(I(I),J(I)) - Z(I(I),J(I)))
1700 GOTO 1740
1710 R(I) = .5*(R(I(I)) + R(I(I)) + 1)
1720 GOTO 1740
1730 P(I) = R(I(I))
1740 NEXT I
1750 RR(H) = R(I(I))
1760 FOR I = 1 TO H
1770 LPRINT "HITS AT H E I J R=1 E 1.000000000000 R(I)

```

```

1780 NEXT I
1790 STOP
1800 -----

1810 -----INITIAL FRINGE CALCULATIONS-----
1820 -----
1830 -----SPAN-TO-INTERVAL INDEXING-----
1830 PRINT "THE SPAN BOUNDARIES ARE LOCATED AT I = 1, I0 + 1, AND AT"
1840 -----I = K - 1 FOR THE FOLLOWING LOOP-----
1850 FOR I = 1 TO K0-1
1860 INPUT "I3(K-1) = "; I3(I)
1870 NEXT I
1880 K = 0
1890 U2 = 1
1900 -----I = INTERVAL FOR FOLLOWING LOOP-----
1910 FOR I = 1 TO I3(K0 - 1)
1920 IF I < I3(U2) THEN 1940
1930 U2 = U2 + 1
1940 K(I) = U2
1950 NEXT I
1960 FOR I = I3(K0 - 1) + 1 TO I0
1970 K(I) = K0
1980 NEXT I
1990 -----
2000 PRINT "THE LEFT-HAND SPAN BOUNDARIES ARE AT RADII" "-----R2(1)=3-----"
2010 FOR I = 1 TO K0
2020 INPUT "R2(K) = "; R2(I)
2030 NEXT I
2040 R2(K0+1) = R1(I0+1)
2050 -----
2060 PRINT "THE INITIAL LOWER FRINGE NUMBERS ARE"
2070 FOR I = 1 TO K0
2080 INPUT "M1(K) = "; M1(I)
2090 NEXT I
2100 -----
2110 -----FIND L,K,N,M1 AT HITS-----
2120 -----
2130 PRINT "THE LEFT-SIDE AXIAL POSITIONS (NEG.) OF THE LOWER INITIAL FRINGES
ARE"
2140 FOR I = 1 TO K0
2150 INPUT "Z2(K) = "; Z2(I)
2160 NEXT I
2170 -----
2180 PRINT "THE RIGHT-SIDE AXIAL POSITIONS (NEG.) OF THE LOWER INITIAL FRINGES
ARE"
2190 FOR I = 1 TO K0
2200 INPUT "Z4(K) = "; Z4(I)
2210 NEXT I
2220 -----
2230 PRINT "THE LEFT-SIDE AXIAL POSITIONS (POS.) Z3(K) OF THE UPPER INITIAL
FRINGES ARE"
2240 FOR I = 1 TO K0
2250 INPUT "Z3(K) = "; Z3(I)
2260 NEXT I
2270 -----
2280 PRINT "THE RIGHT-SIDE AXIAL POSITIONS (POS.) OF THE UPPER INITIAL FRINGES
ARE"
2290 FOR I = 1 TO K0
2300 INPUT "Z5(K) = "; Z5(I)
2310 NEXT I
2320 -----
2330 -----FIND INITIAL FRINGE FRACTION AT EACH HIT-----
2330 -----W1(K) AND W2(K) ARE AXIAL POSITIONS OF LOWER AND UPPER
FRINGES AT A HIT, WHILE S1 AND S2 ARE THE FRINGE SLOPES-----
2340 -----
2350 F(1) = -Z2(1)/(Z3(1) - Z2(1))

```

```

2360 FOR I = 2 TO H-1
2370 S(I) = (Z4(K(I(I)))) - Z2(K(I(I)))/(R2(K(I(I)) + 1) - R2(K(I(I))))
2380 S2(I) = (Z5(K(I(I)))) - Z3(K(I(I)))/(R2(K(I(I)) + 1) - R2(K(I(I))))
2390 W(I) = Z4(K(I(I))) - S(I)*(R2(K(I(I)) + 1) - R(I))
2400 W2(I) = Z5(K(I(I))) - S2(I)*(R2(K(I(I)) + 1) - R(I))
2410 F(I) = -W(I)/(W2(I) - W(I))
2420 NEXT I
2430 F(H) = (1 - S0(I0))/2
2440 FOR I = 1 TO H-1
2450 PRINT "H,S1,S2,W1,W2=";I;S1(I);S2(I);W1(I);W2(I)
2460 NEXT I
2470 STOP
2480 -----
2490 -----FIND THE PHASE SHIFT P(I) AT EACH HIT-----
2500 FOR I = 1 TO H-1
2510 P(I) = 2*(N(L(I(I)))) + (J(I) - E(I))/10 - 2*(M1(K(I(I)))) + F(I)
2520 NEXT I
2530 P(H) = 0
2540 STOP
2550 -----
2560 -----SORT AND INDEX FOR INCREASING RADIUS-----
2570 FOR I = 1 TO H
2580 S(I) = I
2590 RR(I) = R(I)
2600 NEXT I
2610 NR = H
2620 IS = 0
2630 NR = NR - 1
2640 FOR I = 1 TO NR
2650 IF R(I) < R(I+1) GOTO 2730
2660 IS = I
2670 RH = R(I+1)
2680 R(I+1) = R(I)
2690 R(I) = RH
2700 SH = S(I+1)
2710 S(I+1) = S(I)
2720 S(I) = SH
2730 NEXT I
2740 IF IS = 1 GOTO 2620
2750 -----FINAL RESULTS-----
2760 -----
2770 FOR I = 1 TO H
2780 LPRINT "H N,M1 F=";S(I),N(L(I(S(I))));M1(K(I(S(I))));F(S(I))
2790 NEXT I
2800 STOP
2810 FOR I = 1 TO H
2820 LPRINT "H,E,I,J,R,P=";S(I);E(S(I));I(S(I));J(S(I));RR(S(I));P(S(I))
2830 NEXT I
2840 STOP
2850 -----
2860 -----DATA-----
2870 DATA 1,0,-.19,-1,.4,.4,.35
2880 DATA 2,.87,-.22,-1,.38,.38,.38
2890 DATA 3,1.78,-.3,-1,.4,.4,.4
2900 DATA 4,2.52,-.26,-1,.38,.38,.32
2910 DATA 5,3.63,-.23,-1,.36,.38,.34
2920 DATA 6,5.02,-.25,-1,.36,.35,.30
2930 DATA 7, 6.50,-0.0,-1,.37,.32,.32
2940 DATA 8, 7.10,-.08,-1,.38,.30,.32
2950 DATA 9, 8.40,-.14,-1,.38,.30,.28
2960 DATA 10, 9.02,-0.0,-1,.28,.30,.30
2970 DATA 11, 9.28,-.02,-1,.30,.32,.33
2980 DATA 12, 9.45,-.07,-1,.31,.33,.34
2990 DATA 13, 9.67,-.13, 0,.27,.33,.34
3000 DATA 14, 9.90,-.12, 1,.24,.30,.34
3010 DATA 15,10.18,-.25, 1,.20,.25,.25

```

3020 DATA 16,10.37,-.14, 1,.20,.24,.33  
3030 DATA 17,10.69,-.10, 1,.29,.22,.32  
3040 DATA 18,10.92,-.10, 0,.30,.22,.33  
3050 DATA 19,11.51,-.13,-1,.32,.27,.29  
3060 DATA 20,12.52,-.17,-1,.30,.28,.29  
3070 DATA 21,13.58,-.20,-1,.29,.29,.30  
4000 END

V. Final results, giving hit number (H), straddle index (E), interval index (I), fringelet index (J), and radius (R) and phase (P) at each hit.

H,E,I,J,R,P	1	.5	1	5	0	3.7
H,E,I,J,R,P	2	0	1	5	.2696167	3.678824
H,E,I,J,R,P	3	0	2	6	.9770526	3.560872
H,E,I,J,R,P	4	0	2	7	1.504242	3.523936
H,E,I,J,R,P	5	0	3	7	2.2536	3.199322
H,E,I,J,R,P	6	0	4	6	3.675732	2.396723
H,E,I,J,R,P	7	0	5	6	3.70243	2.385411
H,E,I,J,R,P	8	0	5	7	5.008705	2.031905
H,E,I,J,R,P	9	0	6	8	5.504912	2.021648
H,E,I,J,R,P	10	0	6	9	6.000786	2.011532
H,E,I,J,R,P	11	0	7	0	6.5	2
H,E,I,J,R,P	12	0	7	2	6.997422	2.174922
H,E,I,J,R,P	13	0	8	3	7.462748	2.164368
H,E,I,J,R,P	14	0	8	4	8.123563	2.065356
H,E,I,J,R,P	15	0	9	5	8.469821	2.108678
H,E,I,J,R,P	16	0	9	6	8.58581	2.256195
H,E,I,J,R,P	17	0	9	7	8.697731	2.405551
H,E,I,J,R,P	18	0	9	8	8.806726	2.556233
H,E,I,J,R,P	19	0	9	9	8.913634	2.707768
H,E,I,J,R,P	20	0	10	0	9.020001	2.853728
H,E,I,J,R,P	21	0	11	2	9.429931	3.074239
H,E,I,J,R,P	22	0	12	3	9.543399	3.222897
H,E,I,J,R,P	23	0	12	4	9.657842	3.371112
H,E,I,J,R,P	24	.5	13	5	9.785	3.413574
H,E,I,J,R,P	26	0	14	4	9.919374	3.252772
H,E,I,J,R,P	27	0	14	3	9.996132	3.01804
H,E,I,J,R,P	28	0	14	2	10.05446	2.78712
H,E,I,J,R,P	29	0	14	1	10.12546	2.559523
H,E,I,J,R,P	29	0	15	10	10.18	2.334842
H,E,I,J,R,P	32	0	15	9	10.22922	2.112569
H,E,I,J,R,P	31	0	15	8	10.28099	1.889144
H,E,I,J,R,P	30	0	15	7	10.33225	1.66595
H,E,I,J,R,P	34	0	16	6	10.42388	1.42449
H,E,I,J,R,P	33	0	16	5	10.62143	1.1351
H,E,I,J,R,P	35	.5	17	5	10.805	.9520559
H,E,I,J,R,P	36	.5	18	5	11.215	.765296
H,E,I,J,R,P	37	0	19	5	11.7056	.6408196
H,E,I,J,R,P	38	0	19	6	12.46659	.4937954
H,E,I,J,R,P	39	.5	20	7	13.05	.3273974
H,E,I,J,R,P	40	0	21	7	13.61929	.1674431
H,E,I,J,R,P	41	0	21	8	14.16355	.118928
H,E,I,J,R,P	42	0	21	9	14.7233	6.333351E-02
H,E,I,J,R,P	43	0	21	10	15.3	0

## VI. APPENDIX

Interpolation of the non-uniformly-spaced shifted fringes is accomplished by dividing the axial space between the two fringes which bound the cut, within an interval, into ten non-uniform spaces by introducing decimal "fringelets". The lower bounding fringe is taken as the  $J=0$  fringelet and the upper bounding fringe is taken as the  $J=10$  fringelet. Refer to Fig. 2. Let  $D1(I)$ ,  $D2(I)$ , and  $D3(I)$  denote, respectively, the axial fringe-pair spacing at the (left) interval boundary of the lower, central (bounding the cut), and upper fringe pairs. It is necessary to determine the distance  $D(I,J)$ , for  $J=1$  to  $10$ , between the  $J-1$  and  $J$  fringelets. First require that the sum of the fringelet spacings be the central fringe-pair spacing.

$$\sum_{J=1}^{10} D(I,J) = D2(I) \quad A1$$

Information about the non-uniform adjacent fringe-pair spacings is included by requiring that the lower fringelet spacing  $D(I,1)$  be one-tenth of the average fringe-pair spacing for the lower and

central fringe-pairs and that the upper fringelet spacing  $D(I,10)$  be one-tenth of the average fringe-pair spacing for the upper and central fringe pairs.

$$D(I,1) = (D1(I) + D2(I)) / 20 \quad A2$$

$$D(I,10) = (D2(I) + D3(I)) / 20 \quad A3$$

In order to meet these three requirements,  $D(I,J)$  must be at least quadratic in  $J$ . It is assumed that

$$D(I,J) = A(I) + B(I)(J-1) + C(I)(J-1)^2. \quad A4$$

The coefficients  $A$ ,  $B$ , and  $C$  are determined by using Eq. (A4) in Eqs. (A1-A3). It is immediately seen that  $A(I)$  is  $D(I,1)$  and is thus given by Eq. (A2). The other coefficients are evaluated, after a little algebra, as

$$B(I) = (-19 * G(I) + 9 * H(I)) / 8 \quad A5$$

$$C(I) = (3 * G(I) - H(I)) / 8 \quad A6$$

where  $G(I)$  and  $H(I)$  are defined by

$$G(I) = (D3(I) - D1(I)) / 18\theta \quad A7$$

$$H(I) = (D2(I) - D1(I)) / 3\theta \cdot \quad A8$$

In calculating the intersections of the fringelets with the cut, it is necessary to know the distance  $X(I, J)$  between the lower bounding fringe and the  $J$  fringelet. This distance is the sum from  $J'=1$  to  $J$  of  $D(I, J')$ . Evaluation of the sum shows that

$$X(I, J) = J * (A(I) + (J-1) * V(I, J) / 2) \quad A9$$

$$\text{where,} \quad V(I, J) = B(I) + (2J-1) * C(I) / 3 \cdot \quad A10$$

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